1 REAL NUMBERS

Pythagoras is known for his works on rational numbers. He was an ancient Greek mathematician and philosopher who believed that all numbers could be expressed as a ratio of two integers.

Early mathematicians like Pythagoras and Euclid in ancient Greece had knowledge of rational numbers and their properties. The understanding and development of rational numbers evolved over time through the contributions of various mathematicians across different cultures.

CONCEPT MAP

REAL NUMBERS

RATIONAL NUMBERS

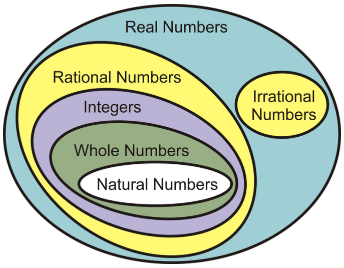
- Rational number between two rational numbers
- Operations on Rational Numbers: Addition, Subtraction, Multiplication, Division
- Represent Rational Numbers on Number Line
- Important Points
  - Multiplicative inverse of a is $\frac{1}{a}$
  - Multiplicative identity is 1
  - Additive identity is 0.
  - Additive inverse of a is -a
  - Absolute value of a number is its numerical value (without sign)

Operation \ Property	Closure	Commutative	Associative	Distributive
Addition	✓	✓	✓	-
Subtraction	✓	×	×	-
Multiplication	✓	✓	✓	✓
Division	✓ (except for 0)	×	×	×

Concept 1

Real Numbers:

The collection of all rational numbers and irrational numbers together is the set of real numbers. It is represented by **R**. A real number is either a rational number or an irrational number.



Rational Numbers:

A number that can be expressed in the form of p/q, where p and q are integers, and q $\neq$ 0, is called a rational number. Rational numbers are commonly called fractions. A rational number can be converted into a decimal number. Similarly, a decimal number can be converted into a rational number.

Example: $\frac{5}{7}, \frac{-3}{11}, \frac{0}{10}$

Misconception :

Misconception: All fractions are rational numbers.

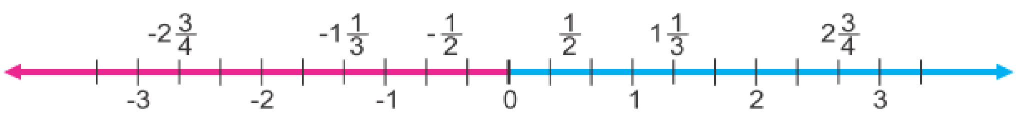
Correction: A fraction is only a rational number, if both the numerator and denominator are integers. For example: $\frac{\pi}{2}$ is not rational.

Note: 0 is a rational number since we can write $0 = \frac{0}{1}$

Representation of Rational Numbers on the Number Line:

The rational number zero is neither negative nor positive. Positive rational numbers are represented to the right of zero on the number line. Negative rational numbers are represented to the left of zero on the number line.

Properties of Rational Numbers:



Property	Operations			
	Addition	Subtraction	Multiplication	Division
Closure	$a + b \in Q$	$a - b \in Q$	$a \times b \in Q$	$a \div b \notin Q$
Commutative	$a + b = b + a$	$a - b \neq b - a$	$a \times b = b \times a$	$a \div b \neq b \div a$
Associative	$a+(b+c) = (a+b)+c$	$a-(b-c) \neq (a-b)-c$	$a\times(b\times c) = (a\times b)\times c$	$a\div(b\div c) \neq (a\div b)\div c$
Distributive	$a\times(b+c) = ab + ac$	$a\times(b-c) = ab - ac$	Not Applicable	Not Applicable
Identity	$a+0 = 0+a = a$	Not Applicable	$a\times1 = 1\times a = a$	Not Applicable

Rational Numbers in Standard Form:

A rational number is said to be in its standard form if its numerator and denominator have no common factor other than 1, and its denominator is a positive integer.

Example: $\frac{-6}{9}$ is $\frac{-2}{3}$, $\frac{-10}{-25}$ is $\frac{2}{5}$

Equivalent Rational Numbers:

If $\frac{p}{q}$ is a rational number and m is any non-zero integer, then $\frac{p}{q} = \frac{mp}{mq}$

In this case, $\frac{p}{q}$ and $\frac{mp}{mq}$ are called equivalent rational numbers.

Example: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \ldots\ldots = \frac{15}{30} = \frac{16}{32} = \ldots. = \frac{144}{288} = \ldots$

Rational Numbers between Two Rational Numbers:

If m and n are two rational numbers such that m < n then $\frac{1}{2}(m + n)$ is a rational number between m and n.

Note: There exist infinitely many Rational Numbers between any two Rational Numbers called as dense property.

Example: Find a rational number lying between $\frac{1}{3}$ and $\frac{1}{2}$.

Solution: **Method-1** The rational number lies between $\frac{1}{3}$ and $\frac{1}{2}$ is

$$\frac{1}{2}\left(\frac{1}{3}+\frac{1}{2}\right) = \frac{1}{2}\left(\frac{2}{6}+\frac{3}{6}\right) = \frac{1}{2}\times\frac{5}{6} = \frac{5}{12} \quad \Rightarrow \frac{1}{3} < \frac{5}{12} < \frac{1}{2}$$

Method-2 Procedure: $\frac{a}{b}$ and $\frac{c}{d}$ are two rational numbers.

Step-1. Make denominators equal in both rational numbers.

Step-2. If we have to find n rational numbers between $\frac{ad}{bd}$ and $\frac{cd}{bd}$, then multiply the numerator and denominator by such a number, so that the difference between the numerator is at least n.

Fun Facts

Rational numbers are like well-behaved students in a classroom- they follow the rule of fractions! But irrational numbers are like rebels- they never end and never repeat!

Method-2: Make the denominator equal by multiplying 2 and 3 are $\frac{1}{3}\times\frac{2}{2}=\frac{2}{6}$ and $\frac{1}{2}\times\frac{3}{3}=\frac{3}{6}$.

To insert 3 rational numbers we multiply the numerator and denominator by such a number so that difference between the numerator is at least 3.

Multiplying the numerator and denominator of both fractions by 4, we get $\frac{8}{24}$ and $\frac{12}{24}$.

Hence the required 3 rational numbers are $\frac{9}{12},\frac{10}{12},\frac{11}{12}$.

Standard Form of Rational Number:

A rational number $\frac{p}{q}$ is said to be in standard form, if q is positive, and p and q have no common divisor other than1.

Method: In order to express a given rational number in standard form, we first convert it into a rational number whose denominator is positive and then we divide its numerator and denominator by their HCF.

Example: Express $\frac{21}{35}$ in standard form:

Solution: The given number is $\frac{21}{35}$

HCF of 21 and 35 is 7

So, we divide its numerator and denominator by 7.

$$\therefore \frac{21}{35}=\frac{21\div 7}{35\div 7}=\frac{3}{5}$$

Hence, $\frac{21}{35}=\frac{3}{5}$ (in standard form).

Riddle

I look bigger but when simplified, I shrink. What am I?

Absolute Value of a Rational Number:

For a rational number x, $|x|=\begin{cases} x, & if \quad x>0 \\ 0, & if \quad x=0 \\ -x, & if \quad x<0 \end{cases}$

Example: The absolute value of $\frac{-15}{17}$ is $\left|\frac{-15}{17}\right|=\frac{15}{17}$

CLASSROOM DISCUSSION QUESTIONS

CDQ 01

1. What is an rational number commonly called?

(A) Whole number

(B) Irrational number

(C) Fraction

(D) Real number

2. How are positive rational numbers represented on the number line?

(A) To the left of zero

(B) To the right of zero

(C) At zero

(D) Between integers

3. When is a rational number said to be in its standard form?

(A) When its numerator is zero

(B) When its denominator is zero

(C) When its numerator and denominator have no common factor other than 1

(D) When its denominator is a negative integer

4. Which property is NOT true for subtraction of rational numbers?

(A) Closure

(B) Associative

(C) Distributive

(D) None

5. What is the lowest form of a rational number?

(A) When its numerator is 1

(B) When its denominator is 1

(C) When its numerator and denominator have a common factor other than 1

(D) When its numerator and denominator have no common factor other than 1

6. When are two rational numbers considered equal?

(A) When their numerators are equal

(B) When their denominators are equal

(C) When their products are equal

(D) When their cross products are equal

7. How many rational numbers are there between any two rational numbers?

(A) One

(B) Two

(C) Infinitely many

(D) None

8. What is an integral part of a decimal number?

(A) The part to the right of the decimal point

(B) The part to the left of the decimal point

(C) The number of digits after the decimal point

(D) The number of digits before the decimal point

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken ☐ Minutes

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6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

Concept 2

Irrational Numbers:

An Irrational Number is a real number that cannot be written as a simple fraction. Irrational means not rational.

Example: π (Pi) is a famous irrational number.

π = 3.1415926535897932384626433832795... (and more)

We cannot write down a simple fraction that equals π.

The popular approximation of 22/7 = 3.1428571428571... is close but not accurate. Another clue is that the decimal goes on forever without repeating, cannot be written as a Fraction. It is irrational because it cannot be written as a ratio (or fraction), not because it is crazy!

So we can tell if it is Rational or Irrational by trying to write the number as a simple fraction.

Example: 9.5 can be written as a simple fraction like this:

9.5 = 19/2

So it is a rational number (Not an irrational)

Note: The best examples for irrational numbers are π, e.

Here are some more examples:

Number	As a Fraction	Rational or Irrational?
1.75	7/4	Rational
0.001	1/1000	Rational
square root of 2	Can't be expressed	Irrational

Other Famous Irrational Numbers:

e **The number *e* (Euler's Number) is another famous irrational number. People have also calculated *e* to lots of decimal places without any pattern showing. The first few digits look like this : 2.7182818284590452353602874713527 (and more ....)**

φ The Golden Ratio is an irrational number. The first few digits look like this : 1.61803398874989484820 .... (and more ....)

√ Many square roots, cube roots, etc are also irrational numbers. Examples :

$\sqrt{3}$	1.7320508075688772935274463415059... (etc)
$\sqrt{99}$	9.9498743710661995473447982100121... (etc)

Terminating & Non-Terminating Decimals:

If a rational number ($\neq$ integer) can be expressed in the form $\frac{p}{2^n \times 5^m}$, where $p \in Z, n \in W$ and $m \in W$ then rational number will be terminating decimal. Otherwise, rational number will be non-terminating recurring decimal.

Example: i) $\frac{3}{8} = \frac{3}{2^3}$ So, $\frac{3}{8}$ is a terminating decimal.

ii) $\frac{7}{250} = \frac{7}{2 \times 5^3}$ So, $\frac{7}{250}$ is a terminating decimal.

iii) $\frac{8}{75} = \frac{8}{5^2 \times 3}$ is a non-terminating recurring decimal, because 3 is the other number in the denominator.

Period and Periodicity:

The recurring part of the non-terminating recurring decimal is called period and the number of digits in the recurring part is called periodicity.

Example: $\frac{1}{3} = 0.\overline{3}$ period = 3, periodicity = 1.

Converting into p/q forms:

Note: **(i)** $a.b\overline{cd} = \frac{abcd - ab}{10^3 - 10^1}$ **(ii)** $a.\overline{bcd} = \frac{abcd - a}{10^3 - 10^0}$

Example: $0.\overline{125} = \frac{125 - 0}{10^3 - 10^0} = \frac{125}{999}$; $3.1\overline{25} = \frac{3125 - 31}{10^3 - 10^1} = \frac{3094}{990}$

Square Root of 5:

Find the square root of 5.

	2 .2 3 6
2	5 .00 00 00
+ 2	-4
42	1 00
+ 2	- 84
443	16 00
+ 3	- 13 29
4466	2 71 00
+ 6	- 2 67 96
4472	3 04

Fun Facts

Irrational numbers are like crazy-artist-they never follow rules but create beautiful patterns!

CLASSROOM DISCUSSION QUESTIONS

CDQ 02

1. Which of the following number is a rational number?

(A) $\sqrt{3}$
(B) 3.5
(C) π
(D) 2.718281828459045....

2. Which of the following number cannot be written as a ratio of two integers?

(A) 4.25
(B) 7
(C) 0.75
(D) $\sqrt{2}$

3. Which of these number is an irrational number?

(A) 0.333...
(B) 22/7
(C) $\sqrt{99}$
(D) 19/2

4. What is the characteristic of an irrational number?

(A) It can be written as a simple fraction.
(B) Its decimal expansion terminates.
(C) Its decimal expansion repeats.
(D) Its decimal expansion goes on forever without repeating.

5. Which of the following is true about the square root of 2?

(A) It is a rational number.
(B) It can be written as a simple fraction.
(C) Its decimal expansion repeats.
(D) It is an irrational number.

6. Which of the following numbers is irrational?

(A) 1.75
(B) 0.001
(C) $\sqrt{2}$
(D) 7/4

7. The number e (Euler's Number) is classified as which type of number?

(A) Rational
(B) Integer
(C) Irrational
(D) Whole number

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Concept 3

Irrational Number Lying Between Two Positive Rational Numbers:

Property: If a and b are two positive rational numbers such that ab is not a perfect square of a rational number, then $\sqrt{ab}$ is an irrational number lying between a and b.

Example: Find two irrational numbers between 2 and 3.

Solution: An irrational number between 2 and 3 is $\sqrt{2\times 3}=\sqrt{6}$.

Similarly an irrational number between 2 and $\sqrt{6}$ = $\sqrt{2\times\sqrt{6}}=\sqrt{2\sqrt{6}}$

$\therefore$ Required numbers are $\sqrt{6}$ and $\sqrt{2\sqrt{6}}$ lie between 2 and 3.

Example: Find three irrationals between 3 and 4.

Solution: An irrational number between 3 and 4 is $\sqrt{12}$

An irrational number between 3 and $\sqrt{12}$ is $\sqrt{3\sqrt{12}}=\sqrt{6\sqrt{3}}$

Another irrational number between $\sqrt{12}$ and 4 is $\sqrt{4\sqrt{12}}=\sqrt{8\sqrt{3}}$

$\therefore$ Required numbers are $\sqrt{6\sqrt{3}}$, $\sqrt{12}$ and $\sqrt{8\sqrt{3}}$ lie between 3 and 4.

Properties of Irrational Numbers:

1. Irrational numbers satisfy the commutative, associative and distributive laws for addition and multiplication.
2. i) Sum of two irrationals need not be an irrational.

   Example: Each one of $(2+\sqrt{3})$ and $(4-\sqrt{3})$ is irrational.

   But, $(2+\sqrt{3})+(4-\sqrt{3})=6$, which is rational.

   ii) Difference of two irrationals need not be an irrational.

   Example: Each one of $(5+\sqrt{2})$ and $(3+\sqrt{2})$ is irrational.

   But $(5+\sqrt{2})-(3+\sqrt{2})=2$, which is rational.

   iii) Product of two irrationals need not be an irrational.

   Example: $\sqrt{3}$ is irrational. But $\sqrt{3}\times\sqrt{3}=3$, which is rational.

   iv) Quotient of two irrationals need not be an irrational.

   Example: Each one of $2\sqrt{3}$ and $\sqrt{3}$ is irrational. But $\frac{2\sqrt{3}}{\sqrt{3}}=2$, which is rational.

Surds:

If 'n' is a positive integer and rational number 'a' (> 0) is not the n^{th} power of any other rational number, then $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ is called a 'surd' or 'radical' of order n and it can be read as n^{th} root of a.

Let $a \in Q^+, n \in I^+$ such that $\sqrt[n]{a}$ is an irrational then $\sqrt[n]{a}$ is called a surd.

Here, $a^{\frac{1}{n}}$ is called an exponential form of a surd and $\sqrt[n]{a}$ is called radical form of a surd.

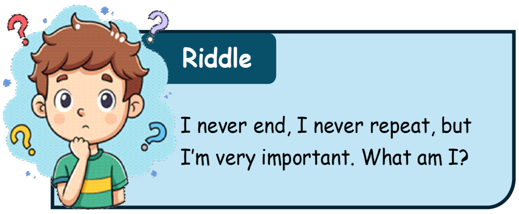
The square roots of numbers that do not have exact square roots are called Surds.

Note: In a surd $\sqrt[n]{a}$,

i) The symbol $\sqrt[n]{\ }$ is called radical sign.

ii) 'a' is called the radicand.

iii) 'n' is called the order of the surd.



Example:

In a surd $\sqrt[3]{5}$, 5 is the radicand, 3 is the order of the surd and $\sqrt[3]{\ }$ is radical sign.

SURDS (RADICALS)	IRRATIONAL NUMBERS
If $n \in I^+$, $a(>0) \in Q$ and 'a' is not n^{th} power of any other rational number then $\sqrt[n]{a}$ is called a surd.	Number which is neither terminating nor repeating decimal.
Examples for surds are $\sqrt{2}, \sqrt{8}, \sqrt[7]{5}, \ldots$	Examples for irrationals are $\sqrt{2}, \sqrt{8}, \sqrt[7]{5}, \pi, e, \ldots$
Every surd is an irrational.	Every irrational number need not be a surd π, e are not surds.

Note: i) $\sqrt{3}, \sqrt[3]{2}, \sqrt[4]{11}, \frac{2}{3}\sqrt[3]{10}$, are surds.

ii) In general, $\sqrt[2]{a}$ is written as $\sqrt{a}$.

iii) The exponential form of $\sqrt{5}$ is $5^{1/2}$ and $\sqrt{5}$ is called the radical form.

iv) 0.5454454. . . is not a surd

v) $\sqrt{2}$ is a surd and also irrational, but π is irrational but not a surd.

vi) $\sqrt{\pi}, \sqrt[3]{e}$ are irrationals but not surds.

Types of Surds Based on Order:

Quadratic surd: A surd of order two is called a quadratic surd.

Example: $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc.

Cubic surd: A surd of order three is called a cubic surd.

Example: $\sqrt[3]{4}, \sqrt[3]{25}, \sqrt[3]{30}$ etc.

Biquadratic surd: A surd of order four is called a biquadratic surd.

Example: $\sqrt[4]{2}, \sqrt[4]{8}, \sqrt[4]{10}$

CLASSROOM DISCUSSION QUESTIONS CDQ 03

1. Which of the following numbers is an irrational number lying between 2 and 3?

(A) 4
(B) 5
(C) $\sqrt{6}$
(D) 3

2. What is an example of a quadratic surd?

(A) $\sqrt[3]{4}$
(B) $\sqrt{2}$
(C) $\sqrt[4]{8}$
(D) 4

3. Find an rational number between 4 and 5.

(A) $\sqrt{16}$
(B) 9/2
(C) $\sqrt{20}$
(D) $\sqrt{21}$

4. Which of the following is a cubic surd?

(A) $3\sqrt{2}$
(B) $\sqrt[3]{3}$
(C) $3\sqrt{25}$
(D) $4\sqrt{2}$

5. What is the radicand in the surd $3\sqrt{5}$?

(A) 3
(B) 5
(C) $\sqrt{5}$
(D) $3\sqrt{2}$

6. Which of the following is an exponential form of a surd?

(A) 5
(B) $5^{\frac{1}{2}}$
(C) $3\sqrt{4}$
(D) $4\sqrt{8}$

7. Which of the following is not a surd?

(A) $\sqrt{2}$
(B) $3\sqrt{8}$
(C) π
(D) $4\sqrt{10}$

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken ☐ Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

Concept 4

Types of Surds Based on Terms:

Simple Surd:

A surd which consists of a single term is called a simple or monomial surd.

Example: $\sqrt[2]{3}, \sqrt[3]{4}, \sqrt[n]{k}$, etc.

Fun Facts

Surds are like mystery numbers- they hide their exact value forever!

Compound Surd:

The sum or difference of a rational number of one or more surd is called a compound surd if the resultant is also a surd.

Example: $3+\sqrt{3},\ 5+\sqrt{3}-\sqrt{7},\ \sqrt{5}+\sqrt{7}-3\sqrt{4}$, etc.

Note: $\pi+\sqrt{2}-\sqrt[3]{5}$ is an irrational number but not a compound surd.

Since $\pi+\sqrt{2}-\sqrt[3]{5}$ is not a surd.

Binomial Surd:

A compound surd consisting of two terms is called a binomial surd.

Example: $\sqrt{2}+\sqrt{5},\ 6-\sqrt{7},\ \sqrt{3}-5$, etc.

Trinomial Surd:

A compound surd consisting of three terms is called a trinomial surd.

Example: $6-\sqrt{3}+\sqrt{5},\ 5+\sqrt{10}+\sqrt[3]{20},\ \sqrt{3}+\sqrt{2}-\sqrt{7}$, etc.

Similar Surds:

If two surds are of different multiples of the same simple surd, then they are called the similar surds or like surds otherwise, they are called dissimilar surds or unlike surds.

Example: **i)** $2\sqrt{5},\ 3\sqrt{5},\ 5\sqrt{5}$ are similar surds or like surds.

ii) $2\sqrt{3},\ 2\sqrt{5},\ 7\sqrt{2}$ are dissimilar surds or unlike surds.

Note: i) The product of two similar quadratic surds is a rational number.

Example: $(3\sqrt{3})(5\sqrt{3})=(3\times5)(\sqrt{3}\times\sqrt{3})=15\times3=45\in Q$

ii) The quotient of two similar surds is a rational number

Example: $7\sqrt[3]{4}\div2\sqrt[3]{4}=\frac{7}{2}\in Q$

Pure Surd:

A surd expressed in the form $a\sqrt[n]{b}$, where a = 1, is called a pure surd or an entire surd.

Example: $\sqrt{6}, \sqrt{20}, \sqrt[3]{5}, \sqrt[3]{25}$ etc.

Mixed Surd:

If 'a' is a non-zero rational number and $\sqrt[n]{b}$ is a monomial surd, then $a \pm \sqrt[n]{b}, a\sqrt[n]{b}$ are called mixed surds.

Example:

$2\sqrt{3}, 4\sqrt[7]{9}, 9-\sqrt{2}, 7+\sqrt[3]{5}$ etc.

Misconception :

Misconception: All square roots are surds.

Correction: $\sqrt{4} = 2$, so it's NOT a surd!

Equal Surd:

Let a, b, c and d are rational numbers. If two surds $a+\sqrt{b} = c+\sqrt{d}$, then a = c and b = d.

Example: If $x+\sqrt{5} = 3+\sqrt{y}$, then x = 3 and y = 5.

Simplest Form of a Surd:

A surd, expressed in the form $a\sqrt[n]{b}$, where 'b' is the least positive rational number, is called the simplest form of the given surd.

Example: **i)** The entire form of $2\sqrt{10}$ is $\sqrt{10\times 2^2} = \sqrt{10\times 4} = \sqrt{40}$

ii) The simplest form of $\sqrt{32}$ is $\sqrt{2\times 16} = 4\sqrt{2}$

Laws of Radicals:

The radicals $\sqrt[n]{a}$, $\sqrt[n]{b}$ are the positive n^{th} root of positive rational numbers a, b respectively for any positive integers m, n, p such that

Riddle

I have a root, but I am not a tree. What am I?

i) $\left(\sqrt[n]{a}\right)^n = \left(a^{\frac{1}{n}}\right)^n = a^{\frac{n}{n}} = a$ ii) $\sqrt[n]{a}\times\sqrt[n]{b} = \sqrt[n]{ab}$ iii) $\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

iv) $\sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a} = \sqrt[n]{\sqrt[m]{a}}$ v) $\sqrt[n]{a^p} = \sqrt[n]{\sqrt[m]{(a^p)^m}} = \sqrt[mn]{a^{pm}}$

CLASSROOM DISCUSSION QUESTIONS

CDQ 04

1. Which of the following is an example of a simple surd?

(A) $2+\sqrt{3}$

(B) $\sqrt{2}$

(C) $3\sqrt{5}-2$

(D) $5+\sqrt{7}$

2. What is a binomial surd?

(A) A surd with one term

(B) A surd with two terms

(C) A surd with three terms

(D) A surd that is a sum of four terms

3. Identify the trinomial surd from the following options.

(A) $\sqrt{3}+5$

(B) $2\sqrt{2}-\sqrt{5}+3$

(C) $2\sqrt{7}$

(D) $\sqrt{2}$

4. Which of the following pair is similar surds?

(A) $\sqrt{2}, 3\sqrt{2}$

(B) $\sqrt{3}, \sqrt{5}$

(C) $3\sqrt{2}, 4\sqrt{3}$

(D) $\sqrt{6}, 2\sqrt{3}$

5. Which of these is an example of a mixed surd?

(A) $\sqrt{2}$

(B) $3\sqrt{5}$

(C) $3+\sqrt{5}$

(D) $\sqrt{9}$

6. What is a pure surd?

(A) A surd expressed in the form $a\sqrt[n]{b}$, where a=1

(B) A surd with more than one term

(C) A surd that can be simplified to a rational number

(D) A surd that is a sum of a rational number and a surd

7. If $\sqrt[3]{4}$ and $\sqrt[3]{5}$ are multiplied, what is the result?

(A) $\sqrt[3]{9}$ (B) $\sqrt[3]{20}$

(C) $\sqrt[6]{20}$ (D) $\sqrt[3]{4+5}$

8. If $2\sqrt{3}$ is divided by $4\sqrt{3}$, what is the result?

(A) $\frac{1}{2}$ (B) 2

(C) 4 (D) $\sqrt{3}$

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken ____ Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

Concept 5

Uses of Laws of Radicals:

By using the laws of radicals, we can

i) Convert a pure surd into a mixed surd

ii) Convert a mixed surd into pure surd

iii) Reduce the given surds to the same order

iv) Compare the given monomial surds.

Fun Facts

Radicals have their own laws-just like a superhero universe!

Convert a Pure Surd into a Mixed Surd:

Example: Express $\sqrt[5]{288}$ as a mixed surd in its simplest form.

Solution: $\sqrt[5]{288} = \sqrt[5]{2^5 \times 3^2} = \sqrt[5]{2^5} \times \sqrt[5]{9} = 2\sqrt[5]{9}$

Example: Express $\sqrt[3]{72}$ as a mixed surd in its simplest form.

Solution: $\sqrt[3]{72} = \sqrt[3]{2^3 \times 3^2} = \sqrt[3]{2^3} \times \sqrt[3]{3^2} = 2\sqrt[3]{9}$

Example: Convert $\sqrt{\frac{50}{4}}$ into its simplest form.

Solution: $\sqrt{\frac{50}{4}} = \sqrt{\frac{25 \times 2}{4}} = \sqrt{\frac{5^2 \times 2}{2^2}} = \sqrt{\left(\frac{5}{2}\right)^2 \times 2} = \frac{5}{2}\sqrt{2}$

Example: Write the simplest form of $\sqrt[3]{24}$

Solution: $\sqrt[3]{24} = \sqrt[3]{8 \times 3} = \sqrt[3]{2^3 \times 3} = 2\sqrt[3]{3}$

Convert a Mixed Surd into a Pure Surd:

Example: Convert $\frac{2}{3}\sqrt{5}$ into pure surd.

Solution: $\frac{2}{3}\sqrt{5} = \sqrt{\left(\frac{2}{3}\right)^2 \times 5} = \sqrt{\frac{4}{9} \times 5} = \sqrt{\frac{20}{9}}$

Example: Express $5\sqrt{6}$ as a pure surd.

Solution: $5\sqrt{6} = \sqrt{5^2 \times 6} = \sqrt{150}$

Misconception :

Misconception: Radicals can be added directly.

Correction: Only like radicals can be added.

Example: **Express $3\sqrt[4]{5}$ as a pure surd.**

Solution: $3\sqrt[4]{5} = \sqrt[4]{3^4 \times 5} = \sqrt[4]{405}$

Reduce the Given Surds to the Same Form of Order:

Example: **Convert the surds $\sqrt[3]{3}$, $\sqrt[4]{4}$ into the same order.**

Solution:

We can write the given two surds into the exponent form as $3^{1/3}$ & $4^{1/4}$.

The exponents $\frac{1}{3}$ and $\frac{1}{4}$ are to be written such that they have a common denominator.

We know that L.C.M of 3 and 4 is 12.

So we can write $\frac{1}{3}$ as $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$ and $\frac{1}{4}$ as $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$

$\therefore \quad 3^{1/3} = 3^{4/12} = (3^4)^{1/12} = \sqrt[12]{81}$ and

$4^{1/4} = 4^{3/12} = (4^3)^{1/12} = \sqrt[12]{64}$

$\therefore \quad \sqrt[3]{3}$ and $\sqrt[4]{4}$ can be written as $\sqrt[12]{81}$ and $\sqrt[12]{64}$

Example: **Convert the surds $\sqrt[4]{10}$, $\sqrt[3]{6}$ and $\sqrt{3}$ into the same order.**

Solution: $\sqrt[4]{10} = 10^{\frac{1}{4}}$, $\sqrt[3]{6} = 6^{\frac{1}{3}}$ and $\sqrt{3} = 3^{\frac{1}{2}}$

Now the L.C.M of 4, 3 and 2 is 12.

$\therefore \quad \sqrt[4]{10} = 10^{\frac{1}{4}} = 10^{\frac{1}{4} \times \frac{3}{3}} = \sqrt[12]{10^3} = \sqrt[12]{1000}$

$\sqrt[3]{6} = 6^{\frac{1}{3}} = 6^{\frac{1}{3} \times \frac{4}{4}} = \sqrt[12]{6^4} = \sqrt[12]{1296}$

$\sqrt{3} = 3^{\frac{1}{2}} = 3^{\frac{1}{2} \times \frac{6}{6}} = \sqrt[12]{3^6} = \sqrt[12]{729}$

$\therefore \quad \sqrt[4]{10}$, $\sqrt[3]{6}$ and $\sqrt{3}$ can be written as $\sqrt[12]{1000}$, $\sqrt[12]{1296}$ and $\sqrt[12]{729}$.

CLASSROOM DISCUSSION QUESTIONS — CDQ 05

1. Express $\sqrt[3]{72}$ as a mixed surd in its simplest form.

(A) $2\sqrt[3]{9}$ (B) $2\sqrt[3]{18}$

(C) $3\sqrt[3]{8}$ (D) $3\sqrt[3]{9}$

2. What is the simplest form of $\sqrt[4]{32}$?

(A) $2\sqrt[4]{4}$ (B) $2\sqrt[4]{2}$

(C) $2\sqrt[4]{8}$ (D) $2\sqrt[4]{16}$

3. Express $4\sqrt{6}$ as a pure surd.

(A) $\sqrt{96}$ (B) $\sqrt{144}$

(C) $\sqrt{150}$ (D) $\sqrt{160}$

4. Reduce the surds $\sqrt[3]{3}$ and $\sqrt[4]{4}$ to the same order.

(A) $\sqrt[6]{9}$ and $\sqrt[8]{16}$

(B) $\sqrt[12]{81}$ and $\sqrt[12]{64}$

(C) $\sqrt[3]{27}$ and $\sqrt[4]{16}$

(D) $\sqrt[9]{27}$ and $\sqrt[4]{16}$

5. Convert $\sqrt[4]{10}, \sqrt[3]{6}$, and $\sqrt{3}$ to the same order.

(A) $\sqrt[12]{10}, \sqrt[12]{6}, \sqrt[12]{3}$

(B) $\sqrt[6]{10^2}, \sqrt[6]{6^3}, \sqrt[6]{3^3}$

(C) $\sqrt[12]{1000}, \sqrt[12]{1296}, \sqrt[12]{729}$

(D) $\sqrt[3]{10^4}, \sqrt[4]{6^3}, \sqrt[2]{3^2}$

6. Which of the following expression $\frac{2\sqrt{5}}{3}$ as a pure surd of ____.

(A) $\sqrt{\frac{20}{9}}$ (B) $\sqrt{10}$

(C) $\sqrt{\frac{40}{9}}$ (D) $\sqrt{30}$

7. If the exponents 1/3, 1/4, and 1/2 are converted to have a common denominator, what is the least common multiple used?

(A) 6 (B) 8

(C) 12 (D) 24

8. Convert $3\sqrt{2}$ into a pure surd.

(A) $\sqrt{9}$ (B) $\sqrt{18}$

(C) $\sqrt{12}$ (D) $\sqrt{6}$

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken ____ Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

Concept 6

Compare the Given Monomial Surds:

Comparison of surds is possible only when they are of the same order. The radicands are then to be compared.

Thus, $\sqrt[3]{5}$ and $\sqrt[3]{7}$ can be compared.

Since the order of the surds are same, now we can compare radicands.

Since 7 > 5 $\Rightarrow$ $\sqrt[3]{7} > \sqrt[3]{5}$.

$\therefore$ $\sqrt[3]{7}$ is greater than $\sqrt[3]{5}$.

Note: In order to compare the surds of different order and different base we first reduce them into same order.

Example: Which is greater $\sqrt[3]{3}$ or $\sqrt[4]{5}$

Solution:

The order of the surds are different. So convert them into the same order.

$\sqrt[3]{3} = 3^{\frac{1}{3}} = 3^{\frac{1}{3}\times\frac{4}{4}} = \sqrt[12]{3^4} = \sqrt[12]{81}$, $\sqrt[4]{5} = 5^{\frac{1}{4}} = 5^{\frac{1}{4}\times\frac{3}{3}} = \sqrt[12]{5^3} = \sqrt[12]{125}$

Since 125 > 81

$\Rightarrow \sqrt[12]{125} > \sqrt[12]{81}$

$\Rightarrow \sqrt[4]{5} > \sqrt[3]{3}$

$\therefore$ $\sqrt[4]{5}$ is greater than $\sqrt[3]{3}$

Example: Arrange the surds in an ascending order of their magnitudes $\sqrt[3]{2}$, $\sqrt[9]{4}$, $\sqrt[6]{3}$.

Solution: The L.C.M of, 3, 9 and 6 is 18.

$\sqrt[3]{2} = 2^{\frac{1}{3}} = 2^{\frac{1}{3}\times\frac{6}{6}} = \sqrt[18]{2^6} = \sqrt[18]{64}$

$\sqrt[9]{4} = 4^{\frac{1}{9}} = 4^{\frac{1}{9}\times\frac{2}{2}} = \sqrt[18]{4^2} = \sqrt[18]{16}$

$\sqrt[6]{3} = 3^{\frac{1}{6}} = 3^{\frac{1}{6}\times\frac{3}{3}} = \sqrt[18]{3^3} = \sqrt[18]{27}$

Since 16 < 27 < 64 $\Rightarrow$

$\sqrt[18]{16} < \sqrt[18]{27} < \sqrt[18]{64} \Rightarrow \sqrt[9]{4} < \sqrt[6]{3} < \sqrt[3]{2}$

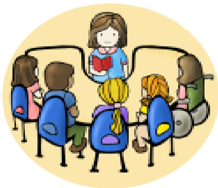
$\therefore$ The ascending order is

$\sqrt[9]{4}$, $\sqrt[6]{3}$ and $\sqrt[3]{2}$.

Misconception :

Misconception: $\sqrt{18}$ is already simplified.

Correction: $\sqrt{18} = 3\sqrt{2}$, so it's NOT simplified!



CLASSROOM DISCUSSION QUESTIONS — CDQ 06

1. Arrange the surds $\sqrt[3]{2}, \sqrt[3]{5}, \sqrt[3]{3}$ in ascending order.

(A) $\sqrt[3]{2}, \sqrt[4]{5}, \sqrt[5]{3}$

(B) $\sqrt[5]{3}, \sqrt[3]{2}, \sqrt[4]{5}$

(C) $\sqrt[4]{5}, \sqrt[3]{2}, \sqrt[5]{3}$

(D) $\sqrt[3]{2}, \sqrt[3]{3}, \sqrt[3]{5}$

2. To compare $\sqrt[3]{4}$ and $\sqrt[4]{3}$, what is the common order used?

(A) 12
(B) 3
(C) 4
(D) 6

3. Which of the following surd is in its simplest form?

(A) $\sqrt[3]{7}$

(B) $\sqrt[4]{16}$

(C) $\sqrt[5]{34}$

(D) $\sqrt[6]{60}$

4. Compare $\sqrt[4]{10}$ and $\sqrt[3]{5}$ by converting to the same order.

(A) $\sqrt[12]{1000} > \sqrt[12]{625}$

(B) $\sqrt[12]{1000} < \sqrt[12]{125}$

(C) $\sqrt[6]{1000} > \sqrt[6]{125}$

(D) $\sqrt[6]{1000} < \sqrt[6]{125}$

5. Which is larger, $\sqrt[3]{6}$ or $\sqrt[4]{7}$?

(A) $\sqrt[3]{6}$

(B) $\sqrt[4]{7}$

(C) They are equal

(D) Cannot be determined

6. Convert the surds $\sqrt[3]{2}$ and $\sqrt[5]{3}$ to the same order.

(A) $\sqrt[15]{32768}$ and $\sqrt[15]{243}$

(B) $\sqrt[15]{8}$ and $\sqrt[15]{243}$

(C) $\sqrt[15]{32}$ and $\sqrt[15]{27}$

(D) $\sqrt[15]{64}$ and $\sqrt[15]{243}$

7. What is the ascending order of $\sqrt[3]{5}$, $\sqrt[4]{4}$, $\sqrt[6]{2}$?

(A) $\sqrt[6]{2}, \sqrt[4]{4}, \sqrt[3]{5}$

(B) $\sqrt[4]{4}, \sqrt[6]{2}, \sqrt[3]{5}$

(C) $\sqrt[3]{5}, \sqrt[6]{2}, \sqrt[4]{4}$

(D) $\sqrt[6]{2}, \sqrt[3]{5}, \sqrt[4]{4}$

8. Convert $\sqrt[3]{3}$ and $\sqrt[4]{2}$ to the same order.

(A) $\sqrt[12]{81}$ and $\sqrt[12]{16}$

(B) $\sqrt[12]{27}$ and $\sqrt[12]{16}$

(C) $\sqrt[12]{9}$ and $\sqrt[12]{4}$

(D) $\sqrt[12]{81}$ and $\sqrt[12]{8}$

MARK YOUR ANSWERS WITH PEN ONLY. Time Taken ____ Minutes

1 Ⓐ Ⓑ Ⓒ Ⓓ	2 Ⓐ Ⓑ Ⓒ Ⓓ	3 Ⓐ Ⓑ Ⓒ Ⓓ	4 Ⓐ Ⓑ Ⓒ Ⓓ	5 Ⓐ Ⓑ Ⓒ Ⓓ
6 Ⓐ Ⓑ Ⓒ Ⓓ	7 Ⓐ Ⓑ Ⓒ Ⓓ	8 Ⓐ Ⓑ Ⓒ Ⓓ	9 Ⓐ Ⓑ Ⓒ Ⓓ	10 Ⓐ Ⓑ Ⓒ Ⓓ

C.D.F. (Concepts, Definitions and Formulae)

1. **Rational Numbers:** A Rational Number can be written in the form of $\frac{p}{q}(q \neq 0)$ where $p,q \in Z$. It is denoted by Q.

2. **Irrational Numbers:** An Irrational Number is a real number that cannot be written as $\frac{p}{q}(q \neq 0)$ where $p,q \in Z$. It is denoted by Q'.

3. **Surds:** If 'n' is a positive integer and a rational number a (> 0) is not the n^{th} power of any other rational number, then $\sqrt[n]{a}$ or $a^{\frac{1}{n}}$ is called a 'surd' or 'radical' of order n and it can be read as n^{th} root of a.

4. **Types of surds based on Order:** The order of a surd indicates the index of root to be extracted. In $\sqrt[n]{a}$, n is called the order of the surd and a is called the radicand.

5. **Pure surd:** A surd of one term which has unity only as rational factor, the other factor being irrational, is called a pure surd.

   Examples: $\sqrt{3}$, $\sqrt[3]{12}$, $\sqrt[7]{16}$ etc. are pure surds.

6. **Mixed surd:** A surd of one term which has a rational factor other than unity, the other factor being irrational, is called a mixed surd.

   Examples: $2\sqrt{3}$; $-3\sqrt{7}$, $5\sqrt[3]{12}$, $\frac{7}{9}\sqrt[4]{8}$ etc. are mixed surds.

7. **Compound surd:** A surd obtained by joining two or more than two pure or mixed surds by + or – signs, is called a compound surd.

   Examples: $\sqrt{2}+\sqrt{3}$; $4\sqrt{5}-\sqrt{3}$; $\sqrt{6}-\sqrt[4]{3}+5\sqrt{7}$ etc. are compound surds.

8. **Similar surds:** If two surds have the same irrational factor, then they are said to be similar surds

   Examples: $\sqrt{3}$, $2\sqrt{3}$, $-\frac{5}{3}\sqrt{3}$ are all similar surds.

9. **Dissimilar surds:** If two surds have different irrational factors, then they are said to be dissimilar surds.

   Examples: $\sqrt{2}$, $\sqrt{3}$, $\sqrt[3]{2}$ are dissimilar surds.

Advanced Worksheet

LEVEL 1

Single Correct Answer Type (S.C.A.T.):

1. While converting the rational number into decimal the process of division terminates, then the decimal is ____

(A) Terminating
(B) Non-terminating
(C) Repeating
(D) Non-terminating not repeating

2. The $\frac{m}{n}$ form of $0.\overline{437}$ is

(A) $\frac{437}{999}$ (B) $\frac{437}{900}$

(C) $\frac{437}{990}$ (D) $\frac{437}{909}$

3. A rational number between $\frac{1}{4}$ and $\frac{1}{3}$ is

(A) $\frac{7}{24}$ (B) 0.29

(C) $\frac{13}{48}$ (D) All of these

4. If $\frac{3}{5} < y < \frac{7}{10}$, which can be a value of y?

(A) $\frac{13}{20}$ (B) $\frac{27}{40}$

(C) $\frac{19}{25}$ (D) $\frac{17}{30}$

5. Compare and pick the smallest rational number among: $\frac{7}{-9}, \frac{-5}{8}, -\frac{2}{3}, \frac{-3}{10}$

(A) $-\frac{7}{9}$

(B) $-\frac{5}{8}$

(C) $-\frac{2}{3}$

(D) $-\frac{3}{10}$

6. A rational expression: $\dfrac{\frac{3}{4}}{\frac{9}{16}}$ equals:

(A) $\frac{4}{3}$ (B) $\frac{3}{2}$

(C) $\frac{1}{2}$ (D) $\frac{8}{3}$

7. Which option is always true for irrational numbers x and y?

(A) x + y is irrational

(B) x · y is irrational

(C) x + (-x) is irrational

(D) None of the above is always true

8. Which of the following decimals is necessarily the decimal expansion of an irrational number?

(A) 0.101001000100001000000... where the number of zeros between 1's increases by one each time

(B) 0.101101101101... repeating 101

(C) 0.4141414141... repeating 41

(D) 0.500000...

9. Which of the following shows density of irrationals (pick true statement)?

(A) Between any two rationals there exists at least one irrational

(B) Between any two irrationals there exists no rational

(C) There are finitely many irrationals between 0 and 1

(D) Sum of two irrationals is always irrational.

10. Which of the following is the best reason why $\sqrt{2}$ is irrational?

(A) Its decimal expansion is long

(B) It cannot be written as a ratio of two integers in lowest terms (proof by contradiction)

(C) $\sqrt{2} \approx 1.414$ so not integer

(D) Because 2 is prime

11. A surd equivalent to $5\sqrt{12}$ is

(A) $10\sqrt{3}$ (B) $12\sqrt{5}$

(C) $20\sqrt{3}$ (D) $15\sqrt{4}$

12. Which of the following is irrational but not a surd?

(A) $\sqrt{2}$ (B) $3\sqrt{5}$

(C) $\sqrt[3]{7}$ (D) $1+\sqrt{3}$

13. Which of the following is an irrational number?

(A) $\sqrt{4}$ (B) $\sqrt{5}$

(C) 1.5 (D) $1.\bar{5}$

14. Name the property of multiplication of rational numbers illustrated by the given statement.

$$\frac{7}{4} \times \left(\frac{-8}{3} + \frac{-13}{12}\right) = \frac{7}{4} \times \frac{-8}{3} + \frac{7}{4} \times \frac{-13}{12}$$

(A) Commutativity

(B) Associativity of multiplication

(C) Distributivity of multiplication over addition

(D) None of these

15. The additive inverse of –a/b is _____.

(A) a/b (B) b/a

(C) –b/a (D) –a/b

16. $10\sqrt{3}$ and $11\sqrt{3}$ are ______ surds.

(A) Pure

(B) Similar

(C) Binomial

(D) Dissimilar

17. If a, b are rational numbers then irrational number between a, b: [By defination]

(A) a+b

(B) a×b

(C) $\sqrt{ab}$

(D) None

18. The sum of the additive inverse and multiplicative inverse of 2 is ___.

(A) 3/2

(B) –3/2

(C) 1/2

(D) –1/2

19. The rational number which is not lying between 5/16 and 1/2 is ___.

(A) 3/8

(B) 7/16

(C) 1/4

(D) 13/32

20. Which of the following statements is true?

(A) $\frac{5}{7} < \frac{7}{9} < \frac{9}{11} < \frac{11}{13}$

(B) $\frac{11}{13} < \frac{9}{11} < \frac{7}{9} < \frac{5}{7}$

(C) $\frac{5}{7} < \frac{11}{13} < \frac{7}{9} < \frac{9}{11}$

(D) $\frac{5}{7} < \frac{9}{11} < \frac{11}{13} < \frac{7}{9}$

21. If 24 trousers of equal size can be prepared in 54 metres of cloth, what length of cloth is required for each trouser?

(A) 4/3 m

(B) 9/4 m

(C) 8/9 m

(D) 14/9 m

22. A rational number between 1/4 and 1/3 is:

(A) 7/24

(B) 0.29

(C) 13/48

(D) All of these

23. Simplify:

$$\left(-\frac{7}{18} \times \frac{15}{-7}\right) - \left(1 \times \frac{1}{4}\right) + \left(\frac{1}{2} \times \frac{1}{4}\right)$$

(A) 27

(B) –303/40

(C) 5/11

(D) 17/24

24. The cost of $7\frac{2}{3}$ metres of rope is Rs.$12\frac{2}{4}$. Find its cost per metre.

(A) Rs.$2\frac{30}{47}$ (B) Rs.$3\frac{18}{25}$

(C) Rs.$1\frac{61}{92}$ (D) Rs.$1\frac{29}{46}$

25. What should be subtracted from $\left(\frac{3}{4}-\frac{2}{3}\right)$ to get $\frac{-1}{6}$?

(A) –6/13

(B) 1/4

(C) 2/7

(D) –1/8

26. The standard form of $\frac{-192}{168}$ is _____.

(A) –2/3 (B) –8/7

(C) –1/7 (D) –6/7

27. The product of two rational numbers is –28/81. If one of the number is 14/27 find the other.

(A) 2/5 (B) 8/17

(C) –2/3 (D) –4/3

28. If a = 5, then the value of $-\left(\frac{a+1}{a}\right)$ is:

(A) –6/5 (B) –5/6

(C) 6/5 (D) 5/6

29. The value of x for which the two rational numbers $\frac{3}{7}, \frac{x}{42}$ are equivalent, is:

(A) 18 (B) 15

(C) 12 (D) 10

30. If x = –2/7, then |–x| is equal to ____.

(A) 2/7 (B) –2/7

(C) 0 (D) 1

31. Find (x+y)÷(x–y) if $x=\frac{1}{4}, y=\frac{3}{2}$.

(A) 6/5 (B) 10/3

(C) –5/4 (D) –7/5

32. The number of irrationals in the given list

$\sqrt{3}, \pi, \frac{1}{3}, 0, \sqrt[5]{2}, \frac{22}{7}, \sqrt{36}$ is:

(A) 3 (B) 4

(C) 5 (D) 6

33. The simplest form of $\sqrt[3]{108}$ is:

(A) $5\sqrt[3]{4}$ (B) $3\sqrt[3]{4}$

(C) $3\sqrt[3]{2}$ (D) $10\sqrt[3]{4}$

34. The simplest form of $5\sqrt{\frac{1}{10}}$ is:

(A) $\sqrt{\frac{5}{4}}$ (B) $\sqrt{\frac{2}{4}}$

(C) $\sqrt{\frac{6}{4}}$ (D) $\sqrt{\frac{5}{2}}$

35. The ascending order of $\sqrt[3]{9}, \sqrt[9]{5}, \sqrt[3]{7}$ is:

(A) $\sqrt[3]{9}, \sqrt[9]{5}, \sqrt[3]{7}$ (B) $\sqrt[9]{5}, \sqrt[3]{9}, \sqrt[3]{7}$

(C) $\sqrt[9]{5}, \sqrt[3]{7}, \sqrt[3]{9}$ (D) $\sqrt[3]{7}, \sqrt[9]{5}, \sqrt[3]{9}$

36. If $3-2\sqrt{2}=a+b\sqrt{2}$, then b = _____.

(A) $\sqrt{2}$ (B) 3

(C) –2 (D) 2

37. A non-terminating decimal from the following is:

(A) $\frac{3}{16}$ (B) $\frac{5}{125}$

(C) $\frac{9}{40}$ (D) $\frac{7}{27}$

LEVEL 2

Multi Correct Answer Type (M.C.A.T.):

38. The $\frac{p}{q}$ form of $0.1\overline{25}$ = _____.

(A) $\frac{12.4}{99}$ (B) $\frac{124}{990}$

(C) $\frac{62}{495}$ (D) $\frac{124}{999}$

39. Which of the following are not equivalent to the rational number 23/54?

(A) 69/216

(B) 138/108

(C) 161/54

(D) 207/270

40. Which of these are not reciprocals of rational numbers lying between 1 & 4?

(A) 3/4 (B) 4/3

(C) 5/17 (D) 2/9

41. Which of the following are rational numbers?

(A) $\sqrt{7}$ (B) $\sqrt{49}$

(C) $\sqrt{2}$ (D) $\sqrt{6.25}$

42. Which of the following fractions are terminating decimals?

(A) $\frac{5}{7}$ (B) $\frac{11}{20}$

(C) $\frac{5}{4}$ (D) $\frac{31}{60}$

43. If 0 < a < 1, then the value of $a+\frac{1}{a}$ is:

(A) $a+\frac{1}{a}>2$ (B) $a+\frac{1}{a}<2$

(C) $2<a+\frac{1}{a}$ (D) $2>a+\frac{1}{a}$

44. Which of the following is / are correct?

(A) The product of two rational numbers is rational

(B) The product of two irrationals must be an irrational

(C) The sum of a rational and an irrational is an irrational

(D) The product of a rational and an irrational is rational.

45. Which of the following is a correct statement?

(A) The irrational number between a and b is $\frac{a+b}{2}$

(B) π is an irrational number

(C) The value of $\sqrt{2}$ up to five decimal places is 1.41421.

(D) Irrational numbers are neither terminating nor recurring decimals.

46. Which of the following is/are correct?

(A) $\sqrt{3}$ is a surd and $\sqrt{3}$ is an irrational number

(B) $\sqrt[3]{5}$ is a surd and $\sqrt[3]{5}$ is an irrational number

(C) π is an irrational number, but it is not a surd

(D) $3+\sqrt{2}$ is a rational number

47. Which of the following is/are correct?

(A) $\sqrt[4]{2} < \sqrt[4]{7}$ (B) $\sqrt[4]{7} < \sqrt[4]{5}$

(C) $\sqrt[5]{10} < \sqrt[5]{13}$ (D) $\sqrt[3]{3} > \sqrt[3]{5}$

Comprehension Passage Type (C.P.T.):

PASSAGE - I

Irrational numbers satisfy the commutative, associative and distributive but not closure with respect to addition and multiplication.

48. If $a=8+\sqrt{5}, b=1-\sqrt{5}$, then a + b is:

(A) Rational

(B) Irrational

(C) Surd

(D) Radical

49. If $a=\sqrt{3}, b=4+\sqrt{3}$, then a – b is ________ number.

(A) Surd

(B) Irrational

(C) Rational

(D) Radical

50. If $a=3+\sqrt{7}, b=3-\sqrt{7}$, then a × b is:

(A) Surd

(B) Irrational

(C) Radical

(D) Rational

PASSAGE - II

A surd, expressed in the form $a\sqrt[n]{b}$, where 'b' is the least positive rational number, is called the simplest form of the given surd.

51. The simplest form of $\sqrt[4]{112}$ is:

(A) $2\sqrt[3]{7}$

(B) $3\sqrt[3]{7}$

(C) $2\sqrt[4]{7}$

(D) $3\sqrt[4]{7}$

52. Which of the following surds is not an simplest form?

(A) $\sqrt{102}$

(B) $\sqrt{116}$

(C) $\sqrt{110}$

(D) $\sqrt{118}$

53. The simplest form of $\sqrt{80}$ is:

(A) $4\sqrt{5}$

(B) $2\sqrt{20}$

(C) $8\sqrt{10}$

(D) $5\sqrt{16}$

LEVEL 3

Matrix Matching Type (M.M.T.):

SET-I

COLUMN- I

54. The equivalent rational number of $\frac{4}{5}$

55. The rational number between $\frac{1}{5}$ and $\frac{7}{5}$

56. The order of the surd $\sqrt[5]{9}$ is

57. The value of $\sqrt{25}$

Column - II

(A) 0.8

(B) 5

(C) $\frac{8}{10}$

(D) 4

(E) $\frac{10}{2}$

SET-II

COLUMN-I

58. $\frac{201}{202}$ expressed as a

59. $\frac{201}{256}$ expressed as a

60. The number which lies between $\frac{201}{202}$ and $\frac{201}{256}$ is

61. 7/37 expressed as a

COLUMN-II

(A) Terminating decimal

(B) $\frac{402}{458}$

(C) Non-terminating but repeating decimal

(D) Non-terminating non-repeating

Assertion Reason Type (A.R.T.):

(A) Both Assertion(A) and Reason(R) are correct and reason(R) is the correct explanation of assertion(A).

(B) Both Assertion(A) and Reason(R) are correct but reason(R) is not the correct explanation of assertion(A).

(C) Assertion(A) is correct but Reason(R) is incorrect.

(D) Assertion(A) is incorrect but Reason(R) is correct.

62. Assertion(A): If a is any rational number, a x 1 = 1 x a = a.

Reason(R): 1 is the multiplicative identity of any rational number.

63. Assertion(A): If a, b, c $\in$ Q, then $a-(b-c)=(a-b)-c$.

Reason(R): Subtraction does not exist under Associative property.

64 Assertion(A): One of the rational between $\frac{1}{5}$ and $\frac{1}{4}$ is $\frac{9}{40}$.

Reason(R): If x and y are any two rational numbers such that x < y, then $\frac{1}{2}(x+y)$ is a rational number between x and y such that $x<\frac{1}{2}(x+y)<y$.

65. Assertion(A): $2007\sqrt[3]{2010}$, $2008\sqrt[4]{2010}, 2009\sqrt[5]{2010}$, are similar surds.

Reason(R): If two surds are different multiples of the same surd, then they are similar surds.

Statement Type (S.T.):

(A) Both Statements are true.

(B) Both Statements are false.

(C) Statement-I is true. Statement-II is false.

(D) Statement-I is false, Statement-II is true.

66. Statement-I: The additive identity of rational number is 0.

Statement-II: The additive inverse of a is –a.

67. Statement-I: The reciprocal of x is 1/x.

Statement-II: The multiplicative inverse of a negative rational number is positive rational number.

68. Statement-I: All irrational numbers are surds.

Statement-II: All surds are irrational numbers.

69. Statement-I: Between $\sqrt{10}$ and $\sqrt{11}$ there are infinitely many irrational numbers.

Statement-II: Between any two real numbers, there are infinitely many irrational numbers.

Integer Type Questions (I.T.Q.):

70. The difference of the additive identity and least whole number=____.

71. If $\frac{5\sqrt{5}}{3}=\sqrt{\frac{125}{k}}$ then k = ________.

72. If $\sqrt[3]{2}=\sqrt[n]{64}$, $\sqrt[9]{4}=\sqrt[n]{16}$, $\sqrt[6]{3}=\sqrt[n]{27}$ then $\frac{n}{9}=$__________.